OC3140
HW/Lab 6 Estimation

1. The mean temperature of a random sample of 36 stations is calculated as $26^o C$. Find the 95 % and 99 % confidence intervals for the population mean temperature. Assume that the population standard deviation is $3^o C$.

Solution:

This is a large sample size with know STD (See Ch.6 p7-p8),

$$n = 36,\ \overline{x} = 26,\ s = 3,\ L(m) = \overline{x} \pm z_{\frac{a}{2}} \frac{s}{\sqrt{n}}.$$

For

$$1 - a = 95\%,\ \frac{a}{2} = 0.025,$$

the standard normal distribution table (Ch. 3 p23) shows that

$$z_{0.025} = 1.96,$$

$$L(m) = \overline{x} \pm z_{\frac{a}{2}} \frac{s}{\sqrt{n}} = 26 \pm (1.96 \times \frac{3}{\sqrt{36}}) = 26 \pm 0.98,$$

thus,

$$25.02 < m < 26.98.$$

The 95 % confidence interval for the population mean temperature is [25.02, 26.98] $^o C$.

For

$$1 - a = 99\%,\ \frac{a}{2} = 0.005,$$

the standard normal distribution table (Ch. 3 p23) shows that

$$z_{0.005} = 2.57,$$

$$L(m)=\overline{x}\pm z_{\frac{a}{2}}\frac{s}{\sqrt{n}}=26\pm(2.57\times\frac{3}{\sqrt{36}})=26\pm1.285,$$

thus,

$$24.715<m<27.285.$$

The 99% confidence interval for the population mean temperature is $[24.715,27.285]\ ^{o}C$.

2. How large a sample is required in problem-1 with 95 % confident that our estimate of m is off by less than $0.5^{o}C$?

Solution:

$n=36$, $\overline{x}=26$, $s=3$,

For

$$1-a=95\%,$$

we have

$$\frac{a}{2}=0.025,\quad z_{0.025}=1.96.$$

From

$$z_{\frac{a}{2}}\frac{s}{\sqrt{n}}<0.5,$$

we have

$$n>\left(\frac{z_{\frac{a}{2}}s}{0.5}\right)^{2}=138.29\approx138.$$

The sample size must be larger than 138.

3. The contents of 7 similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2, and 9.6 liters. Find a 95 % confidence interval for the mean of all such containers, assuming an approximate normal distribution.

Solution:

This is a small sample with unknown STD. Compute as the s-statistics (Ch.6 p8-p9),

$$n = 7\,,\ \overline{x} = 10\,,\ 1 - a = 0.95\,,\ \frac{a}{2} = 0.025\,,\ \ s = \sqrt{\frac{1}{n-1}\sum\left(x - \overline{x}\right)^2} = 0.283\,.$$

The t-distribution table (Ch.5 p17) shows that

$$t_{0.025} = 2.447\,,$$

$$L\left(m\right) = \overline{x} \pm t_{\frac{a}{2}}\frac{s}{\sqrt{n}} = 10 \pm (2.447 \times \frac{0.283}{\sqrt{7}}) = 10 \pm 0.2617\,,$$

$$9.7383 < m < 10.2617\,.$$

The interval is [9.7383, 10.2617].

4. A salinity data (in ppt) set contains: 36.4, 36.1, 35.8, 37.0, 36.1, 35.9, 35.8, 36.9, 35.2, and 36.0. Find a 90 % confidence interval for the variance of the salinity, assuming a normal population.

Solution:

Confidence intervals of s^2 follow the c^2 distribution (Ch.5 p8-p12, Ch.6 p9-p11),

$$\overline{x} = 36.12\,,\ n = 10\,,\ d.f. = n - 1 = 9\,,\ s^2 = \frac{1}{n-1}\sum\left(x_i - \overline{x}\right)^2 = 0.2862$$

$$1 - a = 0.9\,,\ \frac{a}{2} = 0.05\,,\ \ c^2\left(0.05, 9\right) = 16.919\,,\ c^2\left(0.95, 9\right) = 3.325\,,$$

$$L\left(s^2\right)=\left(\frac{(n-1)s^2}{c^2_{\frac{a}{2}}},\frac{(n-1)s^2}{c^2_{1-\frac{a}{2}}}\right)=\left(\frac{9\cdot 0.1862}{16.919},\frac{9\cdot 0.1862}{3.325}\right)=(0.1522,0.7747)$$

we have

$$0.1522 < s^2 < 0.7747 \text{ or } 0.39 < s < 0.88 .$$